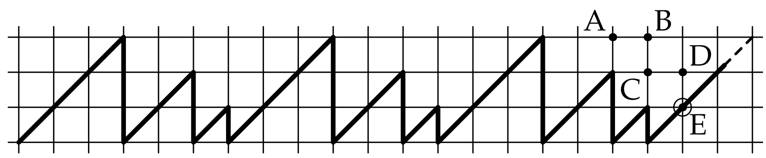
Primary Mathematics Challenge – November 2020

Answers and Notes

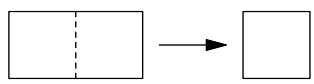
These notes provide a brief look at how the problems can be solved.
There are sometimes many ways of approaching problems, and not all can be given here.
Suggestions for further work based on some of these problems are also provided.

P1 **A**  P2 **D** 40

1	**D**	40	31 July 1980 was $(2020 - 1980)$ years ago.
2	**E**	5 cm	Measure your little finger — it's probably between 5 and 7 cm in length.
3	**A**	impossible	If Grandad were to walk 10 miles in 3 minutes, his speed would be $\frac{60}{3} \times 10 = 200$ mph – surely therefore an impossibility. At a comfortable walking speed, the time taken to walk a mile is somewhere in the range of 17 to 22 minutes. At this rate, 10 miles would take some 200 minutes.
4	**E**	6	The cuts or folds create six equilateral triangles.
5	**A**	11	There are 31 days in August, so the Simpsons will spend the following nights away: 22, 23, 24, 25, 26, 27, 28, 29, 30 and 31 August and 1 September — in total, 11 nights.
6	**B**	58	Spike's haul of slugs is $6 \times 8 + 1 \times 10 = 58$.
7	**A**	3	1 year is roughly 350 days, 2 years roughly 700 days, 3 years roughly 1050, so my sister is now between 2 and 3 years.
8	**D**	45	There are 9 squares on each face, and 5 faces do not have white squares.
9	**B**	15	Postie Patricia delivers to all of the 29 houses from 1 to 29, except the even-numbered ones, 2, 4, 6, . . . 26 and 28, of which there are $28 \div 2 = 14$. So she delivers to $29 - 14 = 15$ houses.
10	**C**	15	Each square contributes 3 edges to the resulting shape, so the polygon will have $3 \times 5 = 15$ edges.
11	**E**	E	As the continued graph below shows, Linus' line passes through point E.



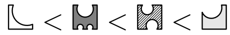
12	**B**	3	It is easy to verify that 2, 4 and 5 are factors of 2020, and that 3 (and hence 6) are not.
13	**E**	red and stripy	If the scarf were blue, then (from Kiran and Manul's suggestions) it would have to be both plain **and** flowery. Thus the scarf is red; now, in order to make one of Lila's claims true, it must also be stripy.
14	**D**	**WGTDP**	Options (a) and (b) lead to taking a shower while fully-dressed; options (c) and (e) suggest waking up would be the last thing on the list. Only option (d) provides the most viable order: wake up, get out of bed, take a shower, dry yourself, put on clothes.
15	**C**	8 mm	The volume of butter is $4 \times 4 \times 4 = 64\,\text{cm}^3$. Spread over an area of $8 \times 10 = 80\,\text{cm}^2$, this will form a layer of butter with a thickness of $64 \div 80 = 0.8$ cm, that is 8 mm.
16	**D**	25	The number of trees planted by each volunteer was $12\,000 \div (32 \times 15)$. Rather than calculating $12\,000 \div 480$, we can simplify to give $3000 \div (8 \times 15) = 200 \div 8 = 25$.
17	**C**	36 cm	Since the folded rectangle is a square with area of $36\,\text{cm}^2$, the sides of the square must be $\sqrt{36} = 6$ cm. Therefore the sides of the rectangle are 12 cm by 6 cm, and so its perimeter is $12 \times 2 + 6 \times 2 = 36$ cm.



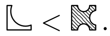
18 **B** 14 The area shaded is simply $25 - 16 + 9 - 4 = 14\,\text{cm}^2$.

19 **A** 18 The number of digital cameras sold fell by 85% to 15% of its 2010 value. Now 15% of 120 000 000 is $\frac{15}{100} \times 120\,000\,000 = 1.5 \times 12\,000\,000 = 18\,000\,000$.

20 **C**  From the first three statements we know that



and, from the fourth, that



As  represents a number smaller than that of each of the four other symbols,  must represent the number five.

21 91 One could approach the question with a systematic search:

number	×3	−4	×5	+6
1	3	$^{-}1$	$^{-}5$	1
2	6	2	10	16
3	9	5	25	31
4	12	8	40	46

So far the final result still has only two digits, though it is noticeable that it is increasing by 15 at each step (perhaps not surprising given that multiplying first by 3 and then by 5 are part of the machine's processing). Continuing we would get … 61, 76, 91 and 106. Hence 91, which is the answer when Marcia puts in 7, is the largest two-digit result.

A second method is to start with the largest two-digit number, 99, and work down. We can certainly see that 96 is 6 greater than a multiple of 5 (90), though $90 + 4$ is not a multiple of 3. The next possible number that is 6 greater than a multiple of 5 (91) does work satisfactorily: $91 = (7 \times 3 - 4) \times 5 + 6$.

The notes below indicate a further algebraic approach to this solution.

22 40 The numbers 1, 2, … , 8, 9 add up to 45. If we add the numbers in the two vertical rectangles and the single horizontal rectangle, the total is $9 + 16 + 25 = 50$. The difference arises from the numbers in the unshaded squares having been counted twice, and so they themselves have a total of $50 - 45 = 5$. Therefore the total of the numbers in the seven shaded squares is $45 - 5 = 40$.

23 18 The total spent, £5, is a multiple of 10p. The cost of a lemon is also a multiple of 10p. It follows that the amount spent on oranges must also be a multiple of 10p. So, because oranges cost 26p each, the number of oranges that Gordon bought was a multiple of 5.

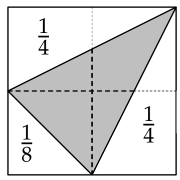
The cost of 5 oranges is 5×30p = £1.50, leaving £$(5 - 1.50)$ = £3.50 to be spent on lemons. But this is not possible as £3.50 is not a multiple of 30p.

The cost of 10 oranges is £2.60, leaving £$(5 - 2.60)$ = £2.40 to be spent on lemons. This is possible as £2.40 = 8×30p. So Gordon could have bought 10 oranges and 8 lemons.

The cost of 15 oranges is £3.90, leaving £$(5 - 3.90)$ = £1.10 to be spent on lemons, which is again impossible.

Since 20 oranges cost £5.20 which is more than £5, we deduce that Gordon bought 10 oranges. We have seen that it follows that Gordon bought 8 lemons, making a total of 18.

24 80 cm The three unshaded triangles take up $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{8}$ of the area of the square, as shown in the diagram on the right. This leaves the fraction shaded as $1 - \frac{1}{4} - \frac{1}{4} - \frac{1}{8} = \frac{3}{8}$, three times the area of the bottom left triangle. Given that the area of the shaded triangle is $150\,\text{cm}^2$, the area of the bottom left triangle is $150 \div 3 = 50\,\text{cm}^2$, and hence the area of the square is $50 \times 8 = 400\,\text{cm}^2$.



Therefore the length of each side of the square is $\sqrt{400} = 20\,\text{cm}$, and so its perimeter measures $4 \times 20 = 80\,\text{cm}$.

25 54 If we begin to compile a "difference table" by subtracting the card number in Alun's arrangement from the equivalent card in Bree's arrangement (as shown here), then we notice that the differences increase by 9 for each column going across, and decrease by 15 for each row going down. This is to be expected because, for each column across, Bree's numbering is increasing by 10 whereas Alun's numbering is increasing by only 1; likewise with the decrease row by row.

0	9	18	27	36	⋯	126	135
-15	-6	3	12	21	⋯	111	120
-30	-21	-12	-3	6	⋯	96	105
⋮	⋮					⋮	⋮
-120	-111	-102	-93	-84	⋯	6	15
-135	-126	-117	-108	-99	⋯	-9	0

Bree – Alun

In order to find the first card for which the numbering has not changed (that is, the difference between Alun's numbering and Bree's numbering is 0), we must go along x columns and down y rows, where $9x - 15y = 0$. Considering 45 as a multiple of both and 9 and 15, it is easy to spot the $x = 5, y = 3$ is the first solution; this is the card with number $5 + 16 \times 3 + 1 = 54$. [The other card not to change its position is 107.]

Some notes and possibilities for further problems

1 The author J.K. Rowling chose Harry Potter's birthday to be 31 July as it was also her own.

7 Pupils might ask when they will be a million hours old (about 114 years). If they don't quite manage that, they should aim at least for a billion seconds (nearly 32 years).

10 Pupils could work out how many edges shapes have for different original regular polygons, starting with a triangle, then a square, a pentagon, a hexagon etc., Can they find a formula giving the number of edges if the number of sides in the original polygon is given?

12 A method for calculating the number of factors a number has (without specifying all of them) is to start by expressing the number as the product of its factors. Let's take 56, as an example. Now we can find that $56 = 2^3 \times 7^1$. Now add 1 to each of the powers in this result, and multiple those answers together: so 56 will have $(3+1) \times (1+1) = 4 \times 2 = 8$ factors. We can quickly find that $2000 = 2^4 \times 5^3$ and so it will have $(4+1)(3+1) = 20$ factors. Similarly 1 billion $= 1\,000\,000\,000 = 2^9 \times 5^9$ has $(9+1) \times (9+1) = 10 \times 10 = 100$ factors. The number of factors for the very next number, 1 000 000 001, which is $7^1 \times 11^1 \times 13^1 \times 19^1 \times 52579^1$ is $(1+1) \times (1+1) \times (1+1) \times (1+1) \times (1+1) = 32$ factors. In the first three examples (56, 2000 and 1 billion) the number of factors is a factor of the original number – maybe pupils could investigate whether it is unusual for the number of factors of a number to divide exactly into the number itself.

13 Sometimes wordy problems like this (where there are *two* categories of information — here, colour and pattern) can be turned into a Carroll diagram (named after Lewis Carroll, alias the nineteenth-century mathematician and logician Charles Dodgson). The diagram for this question would look like this:

	plain	stripy	flowery
red	Kiran		Manul
blue		Lila	

Pupils may recollect other times when it has been helpful to see information represented in a diagram rather than in prose.

14 Pupils might suggest other day-to-day sequences that would have less than satisfactory outcomes if performed in an alternative order.

16 This question was conceived after a news article in the *RSPB* Magazine *Nature's Home* of 10 April 2018 (www.rspb.org.uk/about-the-rspb/about-us/media-centre/press-releases/new-peak-district-woodland-set-to-bring-big-benefits-for-people-and-wildlife/).

21 For an algebraic approach one could start with x, and track it as it works its way through Marcia's machine: $x \to 3x \to 3x - 4 \to 5 \times (3x - 4) \to 5(3x - 4) + 6$. Now this simplifies to $15x - 14$, a number that is 14 less than a multiple of 15. The largest such number with two digits is $105 - 14 = 91$.

25 If we use the observation (made earlier in the answer) that numbers will be in the same place after x columns to the right and y rows down, where $9x - 15y = 0$, we have $9x = 15y$ and hence $3x = 5y$. In other words, the ratio $x : y = 5 : 3$. This leads, in this context, to the (x, y) solutions (0,0), (5,3), (10,6) and (15,9), which in turn indicate the cards numbered 0, 54, 107 and 160. Also, this fixed ratio of $x : y$ accounts for why the unchanged numbers are to found on the diagonal between the first and last.

It is also true, in general, that the number of unchanged numbers is one greater than the highest common factor of x and y, that is, more simply, one greater than the highest common factor of (the number of rows -1) and (the number of columns -1).